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RESEARCH MEMORANDUM

INVESTIGATION OF TURBINES SUITABLE FOR USE IN A TURBOJET
ENGINE WITH HIGH COMPRESSOR PRESSURE RATIO
AND LOW COMPRESSOR-TIP SPEED

IV - EFFECT OF INCREASING BLADE SPEED ON VELOCITY
DIAGRAMS OF TURBINE FOR ENGINE OPERATION AT
CONSTANT ROTATIVE SPEED

By Elmer H. Davison and Robert E. English

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

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**NATIONAL ADVISORY COMMITTEE
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SUMMARY

The range of application of two-stage turbines to driving single-spool compressors was studied by investigating the turbine blade-tip speeds required in order that satisfactory two-stage turbines can be designed to drive a particular compressor over a range of engine operation at rated rotative speed. The relatively low blade-tip speed, high work, and high air flow per unit frontal area of this compressor make critical the problem of designing such a two-stage turbine.

A simplified method for estimating the turbine blade-tip speed required in order to design satisfactory two-stage turbines for this application was developed.

From this investigation, it was concluded that for the purpose of driving a single-spool compressor as part of a turbojet engine to be cruised at rated rotative speed with not less than 0.64 maximum thrust, a two-stage turbine can be designed within the following conditions:

- (1) Relative entrance Mach number to any blade row ≤ 0.80 .
- (2) Turning by any blade row $\leq 118^\circ$.
- (3) There is no deceleration across any blade row.
- (4) The exit tangential velocity is low.
- (5) The exit Mach number is low.

if the turbine characteristics for take-off are within either of the following ranges:

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	Range 1	Range 2
Air flow per unit frontal area, lb/sec-sq ft	≤ 25.8	≤ 23.0
Blade-tip speed, ft/sec	≥ 987	≥ 945
Work input to compressor, Btu/lb	≤ 131	≤ 131
Turbine inlet temperature, $^{\circ}\text{R}$	≈ 2160	≈ 2160

INTRODUCTION

The design of a turbine for a range of turbojet-engine operation rather than for just a single operating condition requires that the turbine-design requirements for the various operating conditions be considered. In reference 1, the design requirements are determined for turbines to drive a particular single-spool, high pressure ratio, low blade-tip speed compressor during engine operation under the following conditions:

- (1) Take-off
- (2) Maximum thrust at altitude
- (3) Altitude cruising
 - (a) With maximum-thrust exhaust-nozzle area and reduced rotative speed
 - (b) At rated rotative speed and increased exhaust-nozzle area
- (4) Engine acceleration at 80 percent equivalent rated speed

For a given ratio of turbine to compressor frontal area, the three compressor parameters that determine turbine-design requirements are mass flow per unit frontal area, work, and blade-tip speed. Increasing compressor work, increasing mass flow per unit compressor frontal area, or decreasing compressor blade-tip speed makes the turbine-design requirements more critical if an attempt is made to stay within a given number of turbine stages, a given turbine frontal area, and preestablished turbine aerodynamic limits. The characteristics of several engines currently under development were investigated; for any given ratio of turbine to compressor frontal area, the requirements of the engine chosen imposed the most severe problem in the design of two-stage turbines because of the relatively high mass flow per unit frontal area, the high work, and the low blade-tip speed of the compressor. If a two-stage turbine can be designed to drive this compressor satisfactorily, two-stage turbines can then be designed to drive all compressors presenting less critical turbine-design requirements.

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The high work output required of a turbine to drive this compressor precluded the possibility of achieving a single-stage turbine design within conventional aerodynamic limits. As shown in reference 2, a two-stage turbine of no greater diameter than the compressor can be designed within conventional aerodynamic limits for engine operation with constant exhaust-nozzle area and variable rotative speed. Reference 3 shows that for engine operation at constant rotative speed and variable exhaust-nozzle area, a two-stage turbine of no greater diameter than the compressor cannot be designed within conventional aerodynamic limits. One way to obtain a satisfactory two-stage design within conventional aerodynamic limits is to increase the turbine blade speed by either increasing the turbine diameter or increasing the turbine rotative speed.

The range of application of two-stage turbines to driving single-spool compressors was therefore investigated at the NACA Lewis laboratory by determining the change in turbine tip speed necessary to obtain satisfactory two-stage turbines designed to drive this particular compressor over a range of engine operation for which the rotative speed is constant and the exhaust-nozzle area is varied. Two possible design changes were considered in this analysis: (1) turbine tip diameter the same as the compressor tip diameter of reference 1 but increased rated rotative speed and (2) unchanged rated rotative speed but turbine tip diameter greater than compressor tip diameter. An estimate was made by an approximate method of the necessary increase in turbine tip diameter or increase in rated rotative speed required in order that satisfactory two-stage turbine designs for engine operation at rated rotative speed could be obtained. Increasing the rated rotative speed of the turbine from that of the compressor in reference 1 would necessitate either redesign of the compressor for the higher rated rotative speed with the same equivalent weight flow and pressure ratio or gearing between the compressor and the turbine.

Two-stage velocity diagrams for turbines to drive the compressor for engine operation at rated rotative speed were investigated for each of the cases considered. The method used to investigate the possible two-stage turbine velocity diagrams was developed in references 2 and 3. From this velocity-diagram study one set of velocity diagrams was selected for presentation for each of the cases considered. These two turbine designs are referred to hereinafter as "increased-diameter turbine" and "increased-speed turbine," respectively.

SYMBOLS

The following symbols are used in this report:

A annular area, sq ft

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a_{cr}	critical velocity, $\sqrt{\frac{2\gamma}{\gamma+1} gRT'}$, ft/sec
D	turbine rotor tip diameter, ft
E	turbine work output, Btu/lb
g	standard acceleration due to gravity, ft/sec ²
$\Delta h_1'$	ideal turbine work, Btu/lb
J	mechanical equivalent of heat, ft-lb/Btu
N	rotative speed, rpm
T	temperature, °R
U	blade speed, ft/sec
V	absolute velocity, ft/sec
W	relative velocity, ft/sec
α	absolute flow angle measured from axial direction, deg
β	relative flow angle measured from axial direction, deg
γ	ratio of specific heats

Subscripts:

u	tangential component
x	axial component
1	entrance to first stator
2	entrance to first rotor
3	entrance to second stator
4	entrance to second rotor
5	exit of second rotor

Superscripts:

'	stagnation or total state relative to a stator
"	stagnation or total state relative to a rotor

ANALYSIS

General Design Considerations

In reference 1, the engine operating conditions for take-off are considered to be a compressor total-pressure ratio of 8.75 and a turbine inlet temperature of 2160° R. At rated speed and pressure ratio, the compressor has an equivalent tip speed of 892 feet per second and an equivalent weight flow of 158 pounds per second. The compressor frontal area is 881 square inches. The turbine-design requirements which must be met by a turbine to drive this compressor at rated rotative speed for both take-off and cruise are shown in reference 1 to be:

	Take-off	Cruise
Work output, Btu/lb	131	115
Percent rated rotative speed	100	100
Turbine inlet temperature, °R	2160	1620
Turbine inlet pressure, lb/sq ft	17,600	6000
Minimum permissible exit annular area, sq in.	383	460
Air flow, lb/sec	158	62.5
Turbine total-pressure ratio	3.42	4.24

The following aerodynamic limits and restriction were used in selecting the velocity diagrams:

(1) Limits

(a) The relative Mach number at the entrance to any blade row should not exceed 0.85.

(b) The amount of turning required of any blade row should not exceed 120°; this applies to the velocity relative to each blade row.

(c) The velocity relative to any blade row should not decrease across the blade row.

(2) Restriction

(a) The tangential component of absolute velocity of the gas leaving the last rotor-blade row should be as near zero as possible.

Limits (a) and (b) are considered the upper limits for which acceptable turbine efficiencies can be expected with present turbine design techniques. The restriction that the exit tangential component of absolute velocity be near zero is particularly important for turbines that are to be operated at rated rotative speed at both take-off and

cruise since, in general, the absolute magnitude of the exit tangential component of velocity will increase under cruising conditions (see reference 3). This could result in a leaving loss, associated with the kinetic energy represented by this velocity, which is appreciable at cruise even though at take-off it is relatively small.

Method of Determining Required Design Changes

An approximate method of analysis was used to predict the design changes required in order to obtain satisfactory two-stage turbine designs which fulfill the turbine-design requirements; this method is presented in the appendix. The nomenclature used in this analysis is shown in figure 1.

For the turbines investigated in references 2 and 3, an assumption of no deceleration in the last rotor and a work division between stages of 70/30 resulted in a good balance between the individual blade rows with respect to the specified velocity-diagram limits and restrictions during take-off operation. These assumptions resulted in the following: The first rotor-blade row was very near the limits on inlet Mach number and turning, the second stator-blade row had high turning with a moderate pressure drop across the blade row, and in accordance with the initial assumption there was no deceleration across the second rotor-blade row. In addition, the axial component of velocity was almost constant across the last rotor and the ratio of hub blade speeds U_5/U_4 was approximately 0.95.

The following conditions were therefore assumed for the analysis presented in the appendix:

- (1) The work division between stages is 70/30.
- (2) The relative velocity is constant across the hub of the last rotor, that is, $W_4 = W_5$.
- (3) The axial velocity is constant across the last rotor, that is, $V_{x,4} = V_{x,5}$.
- (4) The ratio of hub blade speeds U_5/U_4 across the last rotor is 0.95.

Equations (7) and (8) in the appendix are restated here.

$$N = \frac{28.5 \left(V_{u,5} + \sqrt{2.05 g J E_{4,5}} \right)}{\sqrt{\pi^2 D^2 - 4 \pi A_5}} \quad (7)$$

$$D = \frac{1}{\pi} \sqrt{\frac{(28.5)^2 \left(V_{u,5} + \sqrt{2.05 g J E_{4,5}} \right)^2}{N^2} + 4\pi A_5} \quad (8)$$

Equation (7) gives the minimum rotative speed and equation (8), the minimum turbine diameter required to avoid a deceleration across the last rotor.

Equations (7) and (8) have been plotted in figures 2 and 3 for the two design changes considered. These figures were constructed for a 70/30 work division between the two turbine stages. Figure 2 is a plot of turbine rotative speed against exit annular area for constant values of exit tangential component of velocity $V_{u,5}$ and turbine tip diameter. Figure 3 is a plot of turbine tip diameter against exit annular area for constant values of exit tangential component of velocity $V_{u,5}$ and rotative speed.

It is shown in reference 1 that for cruise at rated rotative speed a minimum permissible exit annular area of 460 square inches was required. This minimum permissible exit annular area was assumed in reference 1 to be, for a first approximation, a function of exit axial Mach number alone, that is, independent of engine rotative speed or turbine tip diameter. A vertical line is shown in figures 2 and 3 at the minimum permissible exit annular area of 460 square inches. Exit annular areas to the right of this line (greater than 460 sq in.) will permit cruise at rated rotative speed while exit annular areas to the left of this line (smaller than 460 sq in.) will not. With an exit annular area of 460 square inches the last rotor-blade row would be operating near limiting blade loading (see reference 4) and therefore near the maximum obtainable turbine work output under cruising conditions. For this reason, exit annular areas somewhat greater than 460 square inches should be considered in design in order to permit some margin between the required work output and the maximum obtainable work output at cruise.

Values of exit tangential velocity $V_{u,5}$ less than -100 feet per second were not considered since this tangential component of velocity decreases at cruise and the leaving loss (equal to the ratio of the kinetic energy contained in the tangential velocity at the exit from the last rotor-blade row to the ideal turbine work) becomes appreciable. Values of exit tangential velocity $V_{u,5}$ greater than zero were not considered since they would require a greater increase in turbine rotative speed or turbine tip diameter than is necessary for satisfactory design.

Two designs were chosen from figures 2 and 3 for more detailed study. The points selected for this purpose are shown in figures 2 and 3 by circles labeled "Design conditions" and constitute the following two design conditions:

- (1) Turbine tip diameter, 33.50 inches
 - (a) Exit annular area, 500 square inches
 - (b) Rated rotative speed, 6750 rpm
- (2) Rated rotative speed, 6100 rpm
 - (a) Exit annular area, 500 square inches
 - (b) Turbine tip diameter, 35.50 inches

Velocity-Diagram Calculations

For each of these two design conditions, possible two-stage velocity diagrams for take-off were investigated by means of the one-dimensional scanning method presented in reference 2. For each possible velocity diagram for take-off, the estimated cruise performance of the turbine was checked by employing the one-dimensional cruise analysis developed in reference 3; this cruise analysis determined the magnitude of the leaving loss at cruise and whether or not sufficient turbine work would be produced for cruising operation.

The working charts for this analysis of cruising operation are presented in figures 4 to 7. In figures 4 and 5, the solid lines show for the mean radius the relation between two factors: (1) the relative tangential velocity parameter $(W_u/a_{cr})_5$ at the exit from the last rotor-blade row which must be obtained under cruising conditions if the turbine is to produce enough work to drive the compressor under cruising conditions and (2) the absolute tangential velocity parameter $(V_u/a_{cr})_5$ at the exit from the last rotor-blade row which may prevail under take-off conditions. The dashed line represents the maximum value of relative tangential velocity parameter obtainable within the loading limit in reference 4. Points to the right of the intersection of these two lines represent possible design conditions and those to the left are considered to be outside the range of design. The distance of the dashed line above the solid line is indicative of the margin between design operation for cruise and operation at limiting loading. Figure 4 was prepared for the increased-speed turbine and figure 5, for the increased-diameter turbine. In the same manner, figures 6 and 7 present for the two types of turbine design the leaving loss at cruise as a function of take-off values of the exit tangential velocity parameter $(V_u/a_{cr})_5$ at the mean radius. These figures 4 to 7 are the tools used to compare flow conditions within the turbine for take-off and cruise.

From such a comparison of flow conditions, a design condition was selected for each type of turbine design (increased-rotative-speed and increased-diameter) which is a good compromise of factors affecting the velocity diagrams. For each of these selected design conditions, a set of velocity diagrams was computed which includes radial variations in flow resulting from assumptions of simplified radial equilibrium and free-vortex flow.

RESULTS AND DISCUSSION

The results of the analysis are presented as a set of velocity diagrams and a sketch of the turbine-annulus geometry for each of the increased-speed and increased-diameter designs. The advantage or disadvantage of deviating from these velocity diagrams is discussed in references 2 and 3 and is therefore not presented herein.

Increased Rated Rotative Speed

Design change. - Possible design conditions considering an increased rated rotative speed are shown by the solid lines in figure 2. Any combination of turbine exit annular area and rated rotative speed that falls along these lines represents a design condition for which satisfactory two-stage turbine velocity diagrams for turbines no larger in tip diameter than the compressor of reference 1 may be obtained for engine operation at the rated rotative speed selected. However, for designs having exit annular areas much in excess of the minimum permissible exit annular area (460 sq in.), the blade stress becomes critical. Reference 5 shows that the hub blade stress in the last rotor-blade row due to centrifugal loads varies directly as the product of the exit annular area and the square of the rotative speed. For the design condition investigated herein the untapered blade stress is in the neighborhood of 32,000 pounds per square inch for the blade material considered. Designs having exit annular areas much greater than 500 square inches are probably not practical since the rated rotative speed is shown in figure 2 to increase quite rapidly with increasing exit annular area.

Take-off velocity diagrams. - One set of velocity diagrams for the design condition indicated in figure 2 is shown in figure 8. The velocity diagrams in figure 8 represent a good compromise of the factors affecting velocity diagrams. The turbine configuration for which these diagrams were calculated is shown in figure 9.

The flow parameters of figure 8 for the more critical hub conditions are within the limits specified on inlet Mach number (0.85), turning (120°), deceleration (no deceleration across a blade row), and exit

tangential velocity (exit tangential velocity should be near zero). The exit tangential hub velocity $V_{u,5}$ of -36 feet per second is (within the accuracy of the charts) the same as for the design condition selected from figure 2. From figure 8 it can be noted that the assumption made for the analysis that a 70/30 work division balances the unfavorable flow conditions between the blade rows resulted in hub conditions for the

(1) First stage

(a) Relative rotor inlet Mach number of 0.80

(b) Rotor turning of 118°

(2) Second stage

(a) No deceleration across the rotor

(b) Stator turning of 99° with a moderate pressure drop across the blade row

These conditions are about the optimum that can be obtained unless one stage is penalized in order to improve conditions in the other.

The change in axial component of velocity across the last rotor from 855 to 845 feet per second is in good agreement with the assumption that the axial component of velocity does not change across the last rotor-blade row. In addition, the hub blade-speed ratio across the last rotor-blade row U_5/U_4 of 0.925 is of the order of magnitude assumed, 0.95.

Cruise. - For the take-off velocity diagrams presented in figure 8, the value at the mean radius of the exit tangential velocity parameter $(V_u/a_{cr})_5$ is -0.016. For this take-off value of exit tangential velocity parameter, figure 4 shows that the flow conditions at the exit of the last rotor-blade row are well within the condition of limiting loading and that the turbine will therefore produce the required work for cruise at rated rotative speed. Figure 6 shows that for this take-off value of exit tangential velocity parameter, the leaving loss is 0.006, an insignificant value. An analysis of the change in the turbine velocity diagram from take-off to cruise for a fixed-geometry turbine design made in reference 3 indicates that, as a whole, the flow conditions within the turbine at cruise are at least as favorable as for take-off. For this reason, cruise velocity diagrams were not investigated except to determine that the last rotor-blade row was within limiting loading and that the leaving loss was not significant.

Increased Turbine Tip Diameter

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Design change. - Possible design conditions considering an increased turbine tip diameter are shown by the solid lines in figure 3. Any combination of turbine exit annular area and turbine tip diameter that falls along these lines represents a design condition for which satisfactory two-stage turbine velocity diagrams for turbines with rated rotative speeds the same as that of the compressor of reference 1 (6100 rpm) may be obtained for engine operation at rated rotative speed. In contrast with the designs considered for increased rated rotative speeds, blade stress is not a serious problem inasmuch as rotative speed and mass flow are unchanged; the centrifugal blade stress for the increased-diameter design is 22 percent lower than that for the increased-speed design.

Take-off velocity diagrams. - One set of velocity diagrams for the design condition indicated in figure 3 is shown in figure 10. The velocity diagrams in figure 10 represent a good compromise of the factors affecting the velocity diagrams. The turbine configuration for which these diagrams were calculated is shown in figure 11. The flow parameters of figure 10 for the more critical hub conditions are within the limits specified on inlet Mach number (0.85), turning (120°), deceleration (no deceleration across a blade row), and exit tangential velocity (exit tangential velocity should be near zero).

The exit tangential hub velocity $V_{u,5}$ of -33 feet per second is (within the accuracy of the charts) the same as for the design condition. The change in axial velocity from 868 to 845 feet per second across the last rotor, although in disagreement with the assumption of no change, has a negligible effect on the results of the simplified analysis. The ratio of blade speeds across the hub of the last rotor U_5/U_4 is 0.94 and is in close agreement with the assumed value of 0.95.

Cruise. - For the take-off velocity diagrams presented in figure 10 the value at the mean radius of the exit tangential velocity parameter $(V_u/a_{cr})_5$ is -0.015. For this take-off value of exit tangential velocity parameter, figure 5 shows that the flow conditions at the last rotor-blade exit are well within the condition of limiting loading and that the turbine will therefore produce the required work for cruise at rated rotative speed. Figure 7 shows that for this take-off value of exit tangential velocity parameter the leaving loss is 0.006, an insignificant value. As stated previously in the results for the increased-speed design, an investigation of the complete cruise velocity diagrams was not considered necessary.

General Significance

The results of the analysis, as so far presented, are specific rather than general because they are related to a particular compressor considered as part of a turbojet engine. The value of these results can be increased if, from them, general conclusions can be drawn concerning the entire class of single-spool compressors. In order for the general significance of these results to be apparent, the turbine-design requirements for driving this compressor over a range of engine operation for which the engine rotative speed is constant must be compared with the turbine-design requirements of other single-spool compressors. If a two-stage turbine can be designed to drive this compressor satisfactorily, two-stage turbines can then be designed to drive all compressors presenting less critical turbine design problems. In the following discussion, the turbine inlet temperature for take-off is presumed to be near 2160°R because a large change in temperature will alter the turbine-design requirements.

For the purpose of driving a single-spool compressor as part of a turbojet engine to be cruised at rated rotative speed with not less than 0.64 maximum thrust, a two-stage turbine can be designed within the following conditions:

- (1) Relative entrance Mach number to any blade row ≤ 0.80 .
- (2) Turning by any blade row $\leq 118^{\circ}$.
- (3) There is no deceleration across any blade row.
- (4) The exit tangential velocity is low.
- (5) The exit Mach number is low.

if the turbine characteristics for take-off are within either of the following ranges:

	Range 1 (from increased- rotative-speed design)	Range 2 (from increased- diameter design)
Air flow per unit frontal area, lb/sec-sq ft	≤ 25.8	≤ 23.0
Blade-tip speed, ft/sec	≤ 987	≤ 945
Work input to compressor, Btu/lb . . .	≤ 131	≤ 131

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SUMMARY OF RESULTS

In order to study the range of application of turbines to driving single-spool compressors, an investigation was conducted to determine the increase in turbine blade-tip speed necessary to obtain satisfactory two-stage turbine velocity diagrams for a turbine to drive a particular compressor over a range of engine operation at rated rotative speed. A simplified method of analysis, which accurately predicted the required design changes necessary to obtain satisfactory two-stage turbines for this application, was evolved for this study.

For this particular compressor, which has a tip diameter of 33.50 inches, a rated rotative speed of 6100 rpm, an air flow of 158 pounds per second, and a pressure ratio of 8.75, the following results concerning the turbine were obtained:

1. If the size of the turbine tip diameter was limited to that of the compressor, a change in rated rotative speed from 6100 rpm to 6750 rpm was sufficient to permit design of a satisfactory two-stage turbine for this application.
2. If the rated rotative speed was fixed at 6100 rpm, a turbine tip diameter 2.00 inches in excess of the 33.50-inch compressor tip diameter was sufficient to permit design of a satisfactory two-stage turbine for this application.
3. Both design changes produced satisfactory and very similar velocity diagrams; however, the level of centrifugal turbine-blade stress for the increased-diameter design was approximately 22 percent lower than that for the increased-speed design.

CONCLUSION

For the purpose of driving a single-spool compressor as part of a turbojet engine to be cruised at rated rotative speed with not less than 0.64 maximum thrust, a two-stage turbine can be designed within the following conditions:

- (1) Relative entrance Mach number to any blade row ≤ 0.80 .
- (2) Turning by any blade row $\leq 118^\circ$.
- (3) There is no deceleration across any blade row.
- (4) The exit tangential velocity is low.
- (5) The exit Mach number is low.

if the turbine characteristics for take-off are within either of the following ranges:

	Range 1	Range 2
Air flow per unit frontal area, lb/sec-sq ft	25.8	23.0
Blade-tip speed, ft/sec	987	945
Work input to compressor, Btu/lb	131	131
Turbine inlet temperature, °R	2160	2160

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National Advisory Committee for Aeronautics
Cleveland, Ohio

APPENDIX - METHOD FOR ESTIMATING REQUIRED BLADE SPEED

Assumptions

For the turbines investigated in references 2 and 3, an assumption of no deceleration in the last rotor and a work division between stages of 70/30 was found to make both stages of the two-stage turbines about equally critical with respect to the specified velocity-diagram limits and restrictions during take-off operation. In addition, the axial component of velocity was almost constant across the last rotor and the ratio of hub blade speeds U_5/U_4 was approximately 0.95.

The following conditions were therefore assumed:

- (1) The work division between stages is 70/30.
- (2) The relative velocity is constant across the hub of the last rotor, that is, $W_4 = W_5$.
- (3) The axial velocity is constant across the last rotor, that is, $V_{x,4} = V_{x,5}$.
- (4) The ratio of hub blade speeds U_5/U_4 across the last rotor is 0.95.

Derivation

With the exceptions of tip diameter D and exit annular area A_5 , the symbols used in this derivation refer to conditions at the hub of the last rotor.

For no deceleration across the last rotor,

$$W_4^2 = W_5^2 \quad (1)$$

The vector additions in figure 1 are used to write equation (1) as

$$V_{x,4}^2 + (V_{u,4} - U_4)^2 = V_{x,5}^2 + (V_{u,5} - U_5)^2$$

With

$$V_{x,4} = V_{x,5}$$

this expression reduces to

$$V_{u,4} - U_4 = U_5 - V_{u,5} \quad (2)$$

The Euler work equation

$$E_{4,5} = \frac{U_4 V_{u,4} - U_5 V_{u,5}}{gJ}$$

is combined with equation (2), giving

$$E_{4,5} = \frac{U_4 (U_4 + U_5 - V_{u,5}) - U_5 V_{u,5}}{gJ} \quad (3)$$

Introducing the assumption that

$$U_5 = 0.95 U_4$$

permits equation (3) to be reduced to

$$U_5 = 0.475 \left(V_{u,5} + \sqrt{V_{u,5}^2 + 2.05 gJ E_{4,5}} \right) \quad (4)$$

For amounts of exit tangential velocity $V_{u,5}$ resulting in comparatively small values of leaving loss, equation (4) can be approximated by

$$U_5 = 0.475 \left(V_{u,5} + \sqrt{2.05 gJ E_{4,5}} \right) \quad (5)$$

The rotative speed N and the blade speed U_5 are related in the following way:

$$N = \frac{60 U_5}{\sqrt{\pi^2 D^2 - 4\pi A_5}} \quad (6)$$

Combination of equations (5) and (6) yields

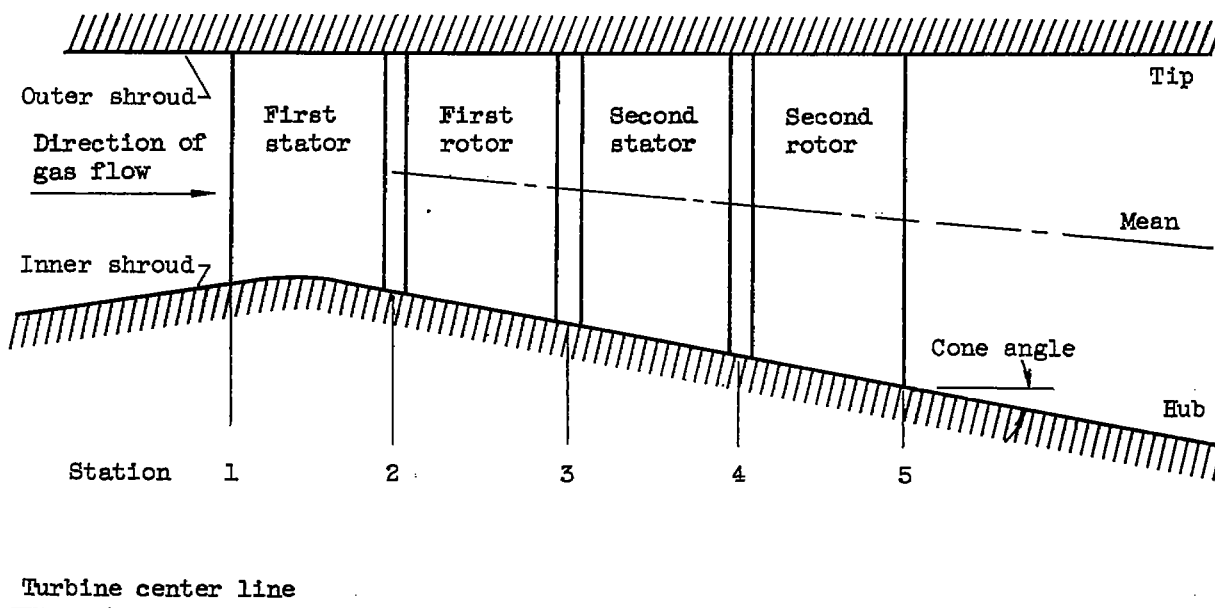
$$N = \frac{28.5 \left(V_{u,5} + \sqrt{2.05 gJ E_{4,5}} \right)}{\sqrt{\pi^2 D^2 - 4\pi A_5}} \quad (7)$$

and

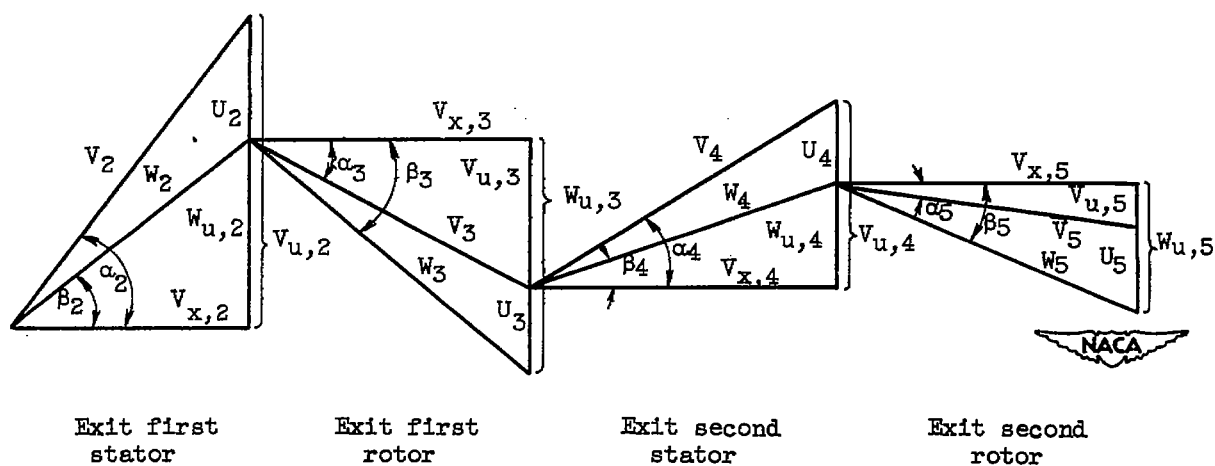
$$D = \frac{1}{\pi} \sqrt{\frac{28.5^2 \left(V_{u,5} + \sqrt{2.05 gJ E_{4,5}} \right)^2}{N^2} + 4\pi A_5} \quad (8)$$

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(a) Turbine nomenclature.



(b) Velocity-diagram nomenclature.

Figure 1. - Turbine and velocity-diagram nomenclature. V_u positive when in same direction as U ; V_u negative when in direction opposite to U .

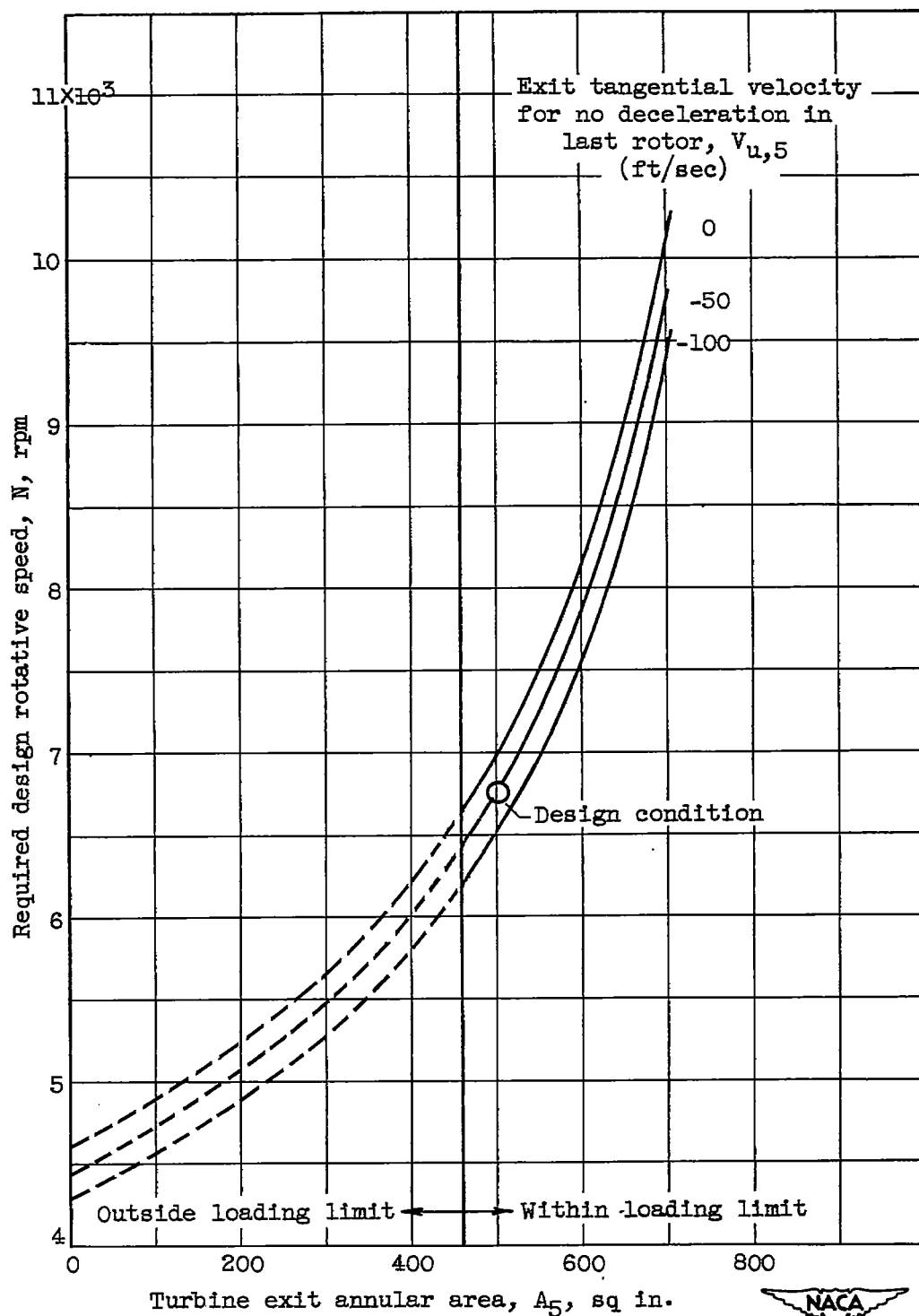


Figure 2. - Turbine-design requirements for turbine with fixed tip diameter.

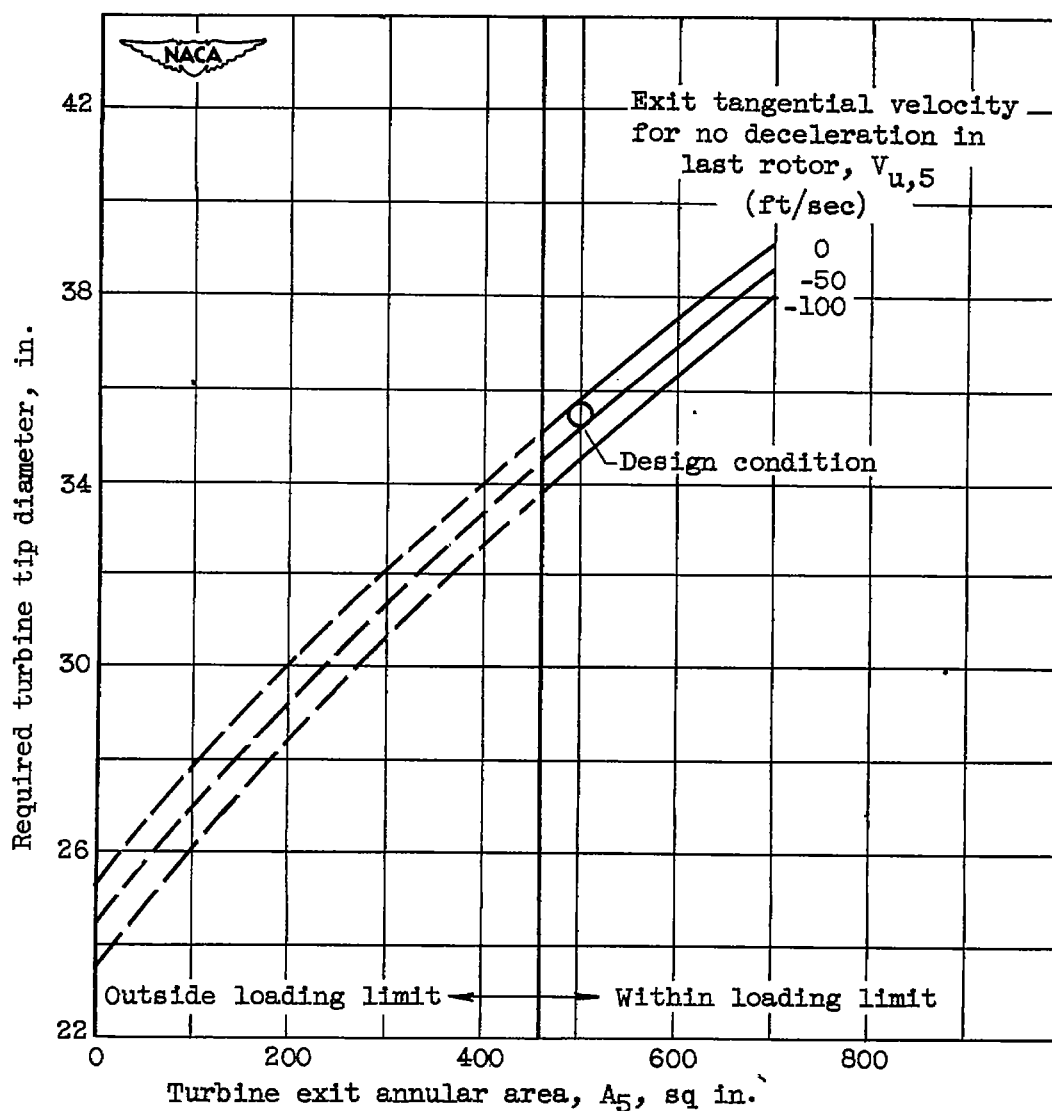


Figure 3. - Turbine-design requirements for turbine with fixed rated rotative speed.

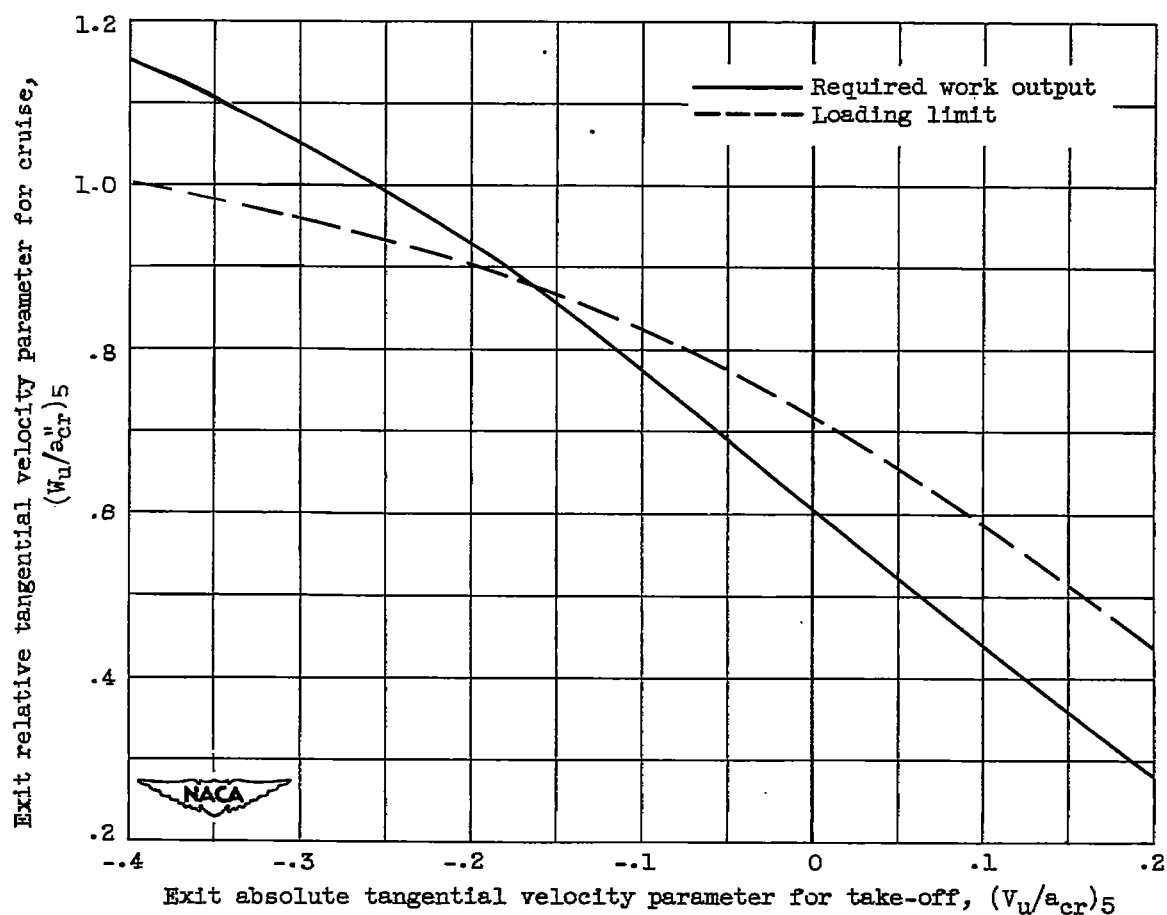


Figure 4. - Comparison of required and limiting exit relative tangential velocity parameter $(W_u/a_{cr}^u)_5$ at cruise for various increased-speed turbines.

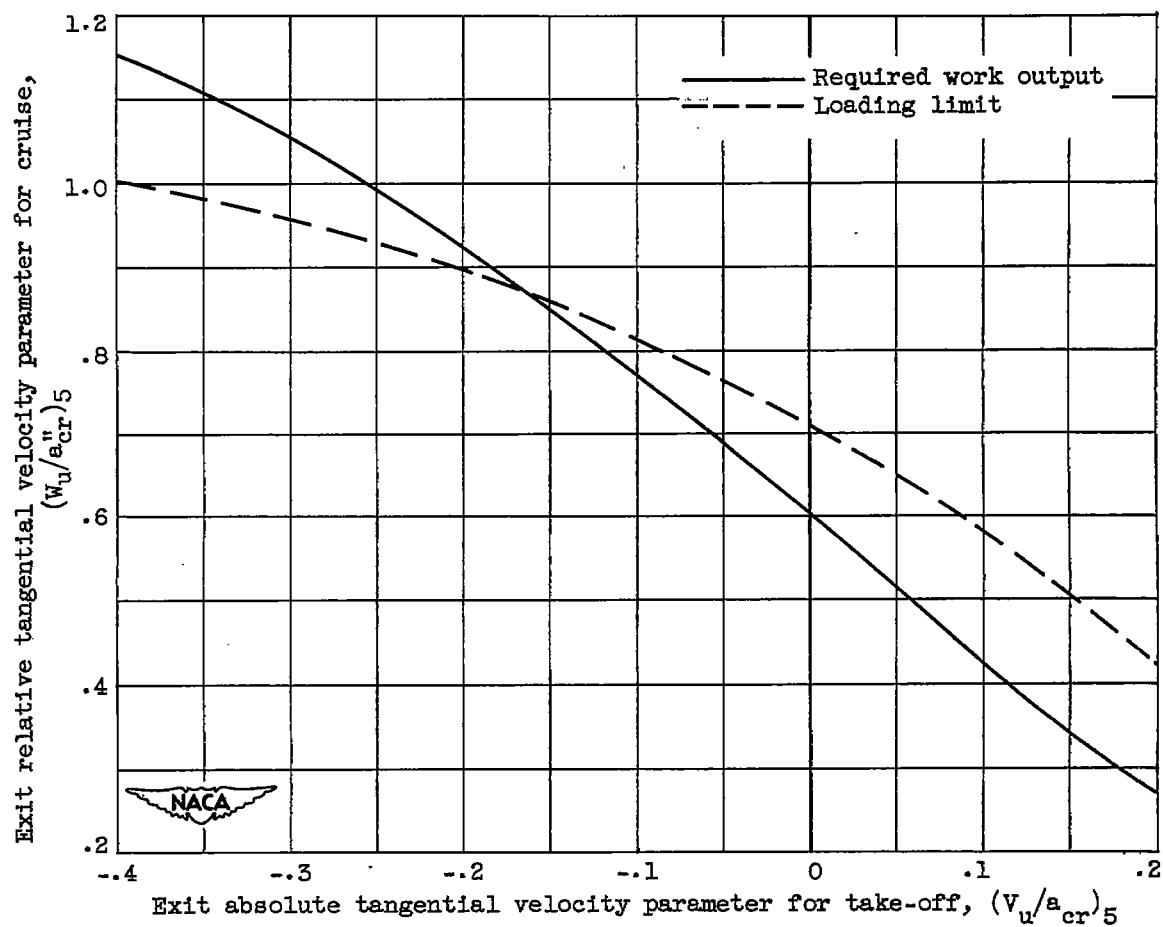


Figure 5. - Comparison of required and limiting exit relative tangential velocity parameter $(W_u/a''_{cr})_5$ at cruise for various increased-diameter turbines.

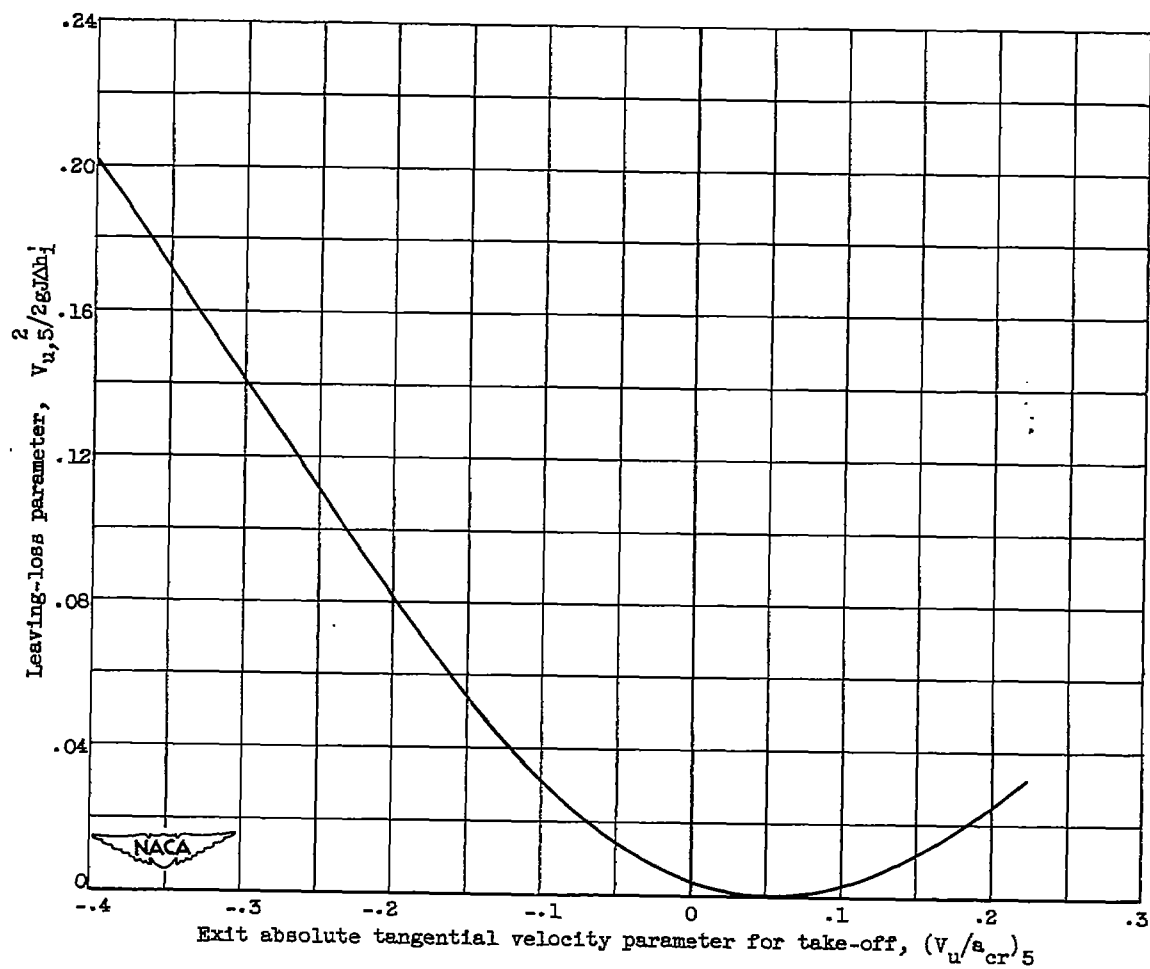


Figure 6. - Leaving loss at cruise for various increased-speed turbines.

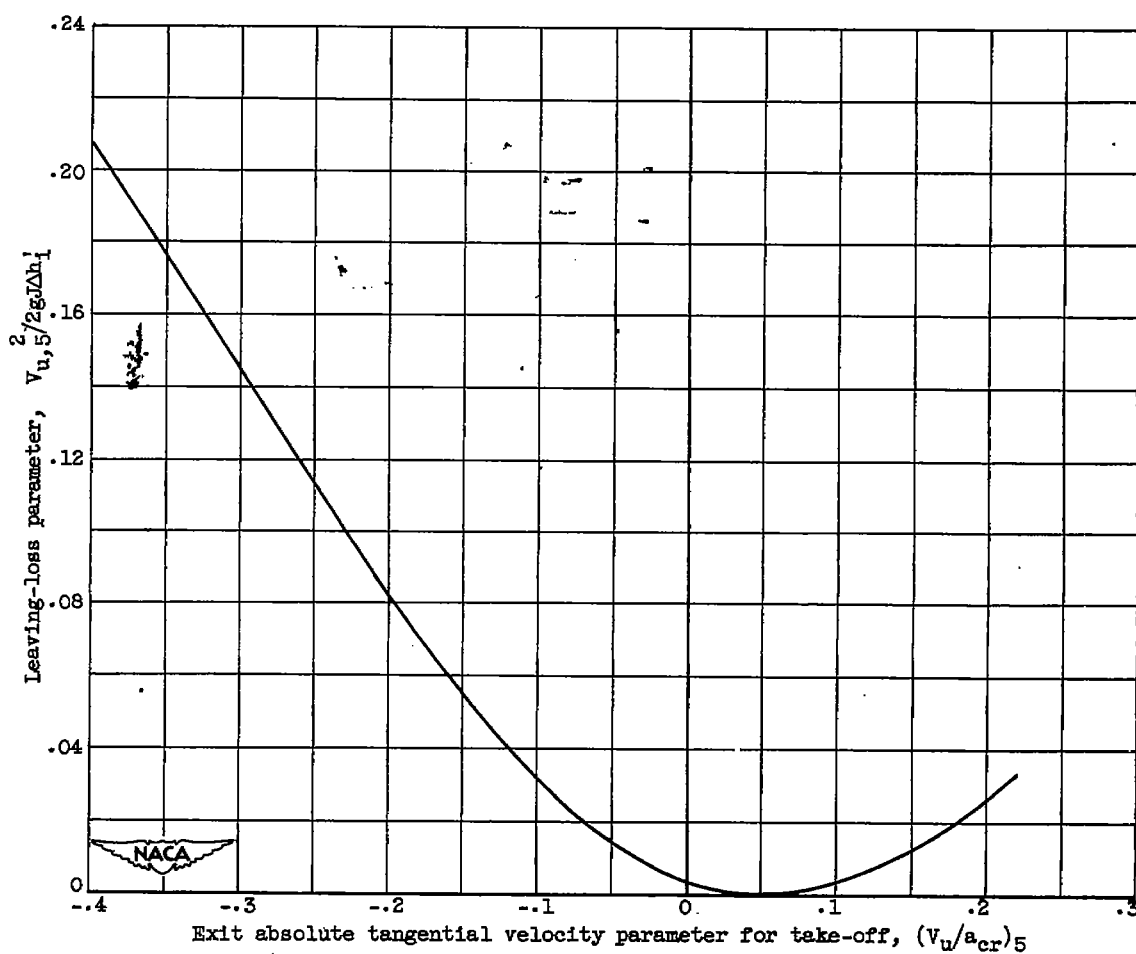


Figure 7 - Leaving loss at cruise for increased-diameter turbines.

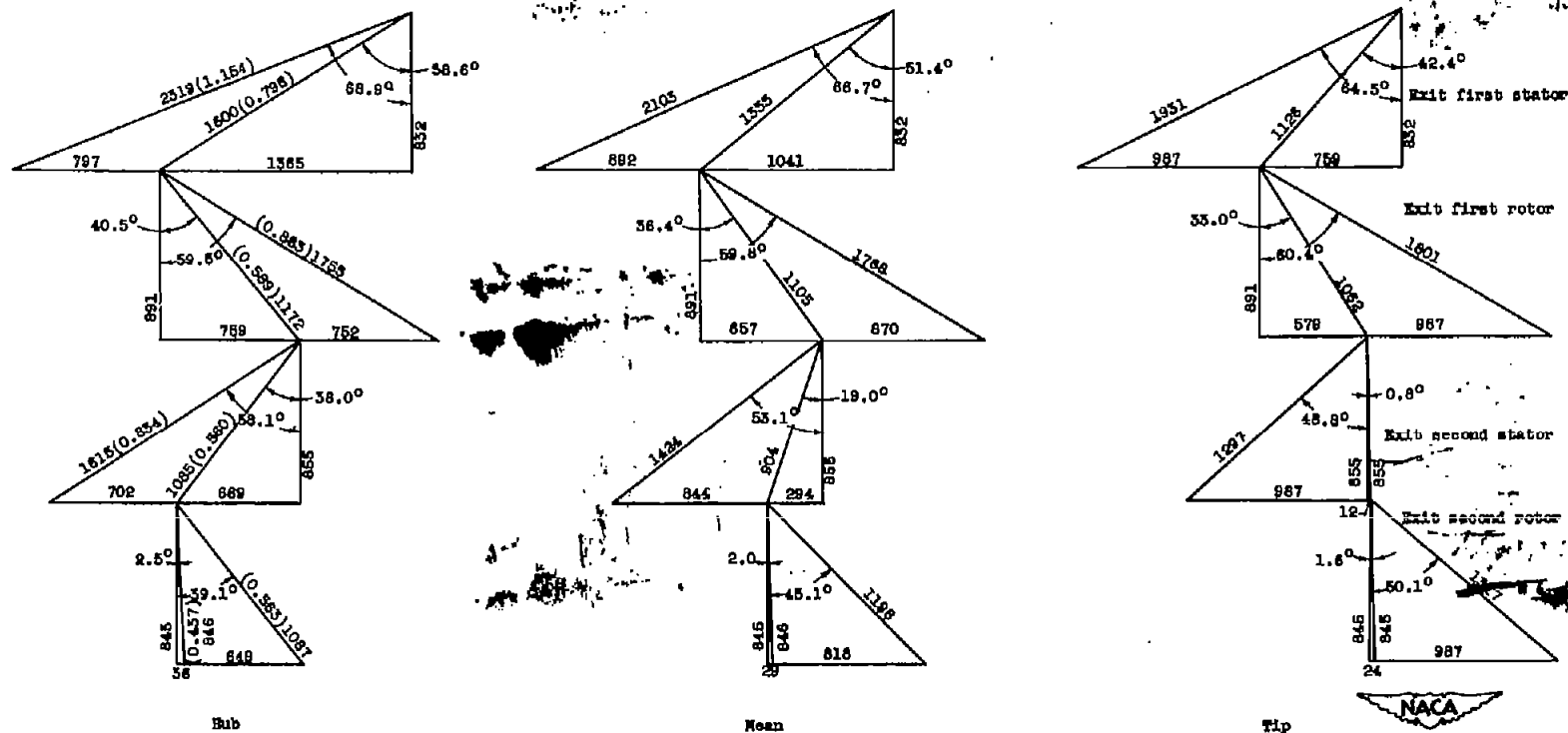


Figure 8. - Take-off velocity diagrams with assumed radial variations in flow for turbine having 70/30 work division, exit annular area of 500 square inches, and rated rotational speed of 6750 rpm. Numbers in parentheses are Mach numbers based on local velocity of sound; velocities are in feet per second; angles are in degrees.

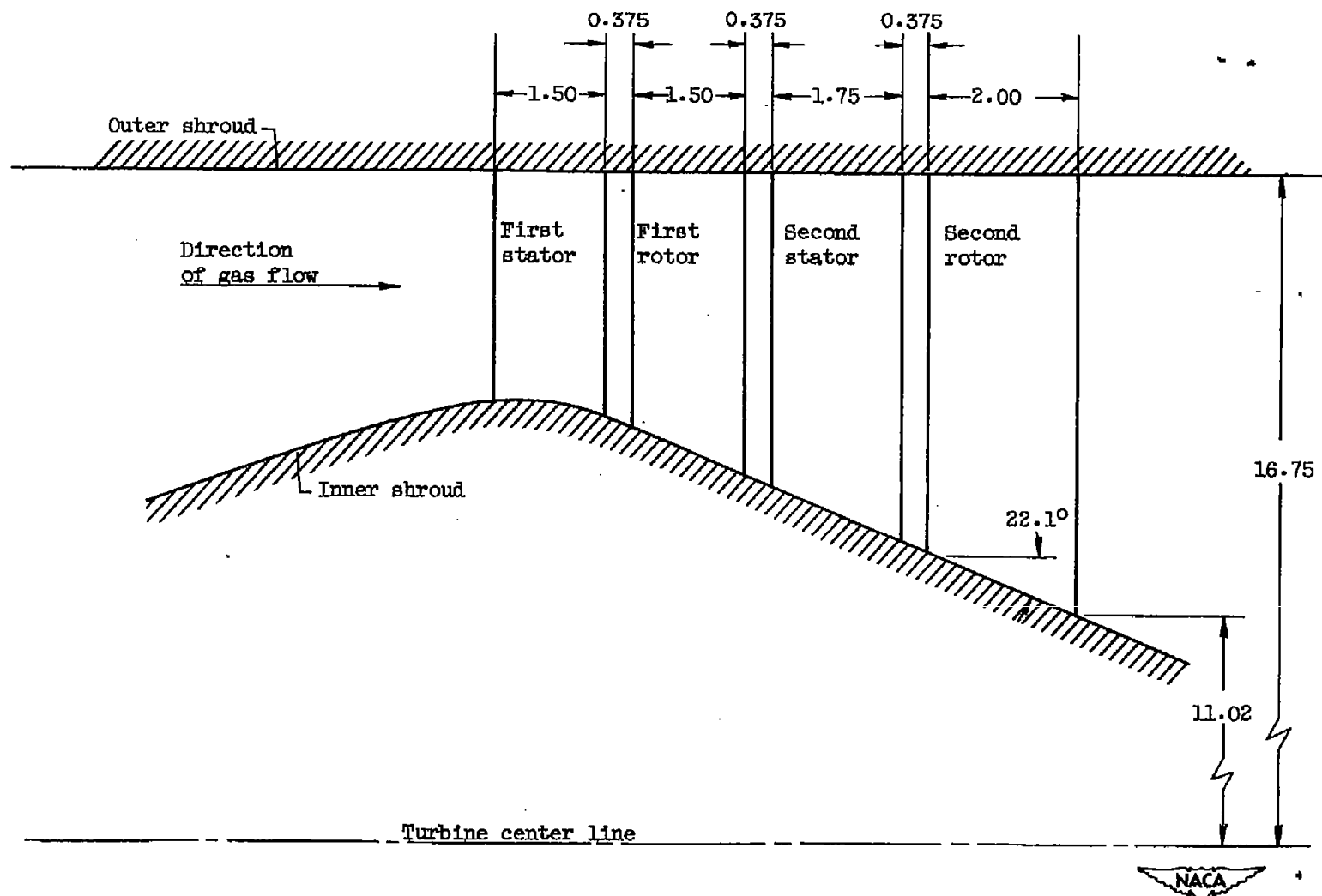


Figure 9. - Two-stage turbine configuration for increased-speed turbine. Dimensions are in inches.

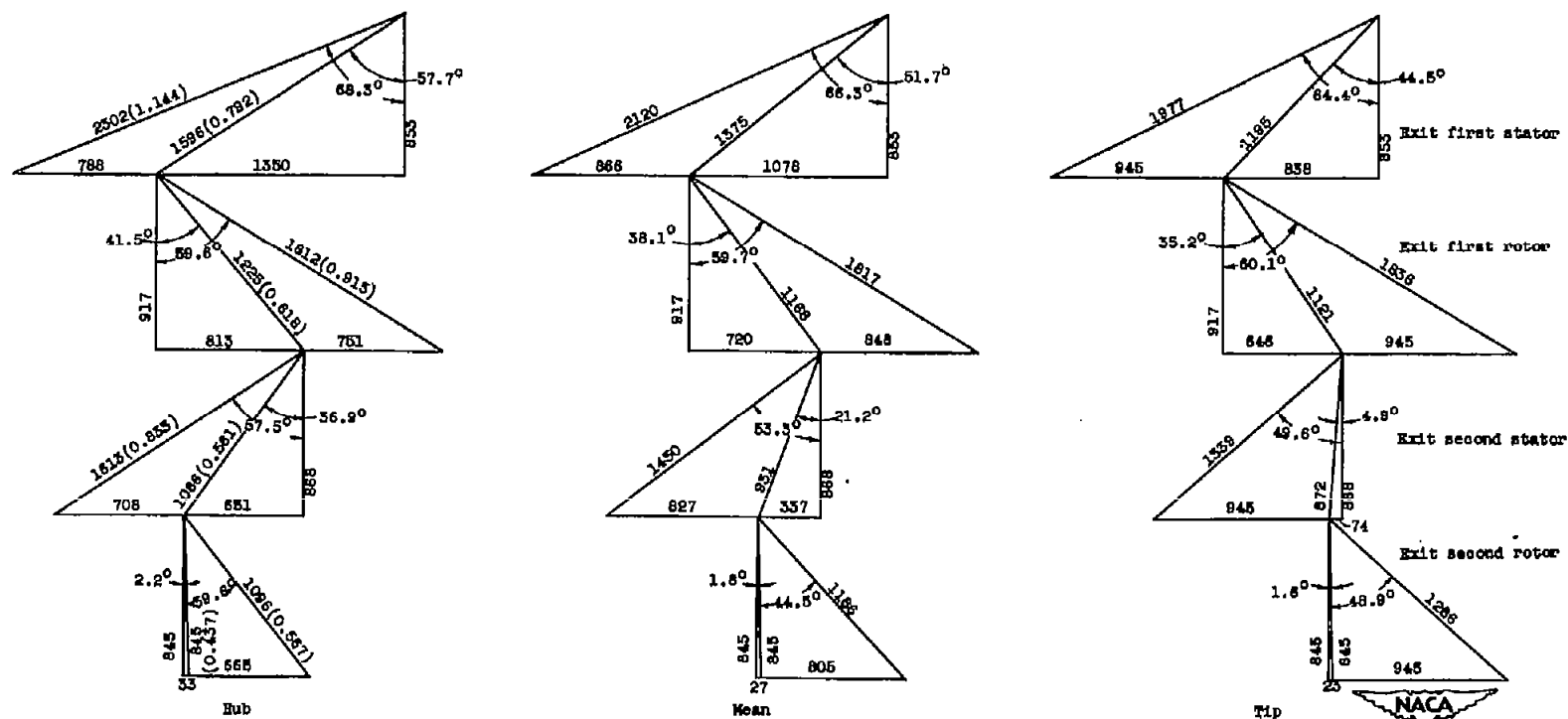


Figure 10. - Take-off velocity diagrams with assumed radial variations in flow for turbine having 70/30 work division, exit angular area of 500 square inches, and turbine tip diameter of 35.50 inches. Numbers in parentheses are Mach numbers based on local velocity of sound; velocities are in feet per second; angles are in degrees.

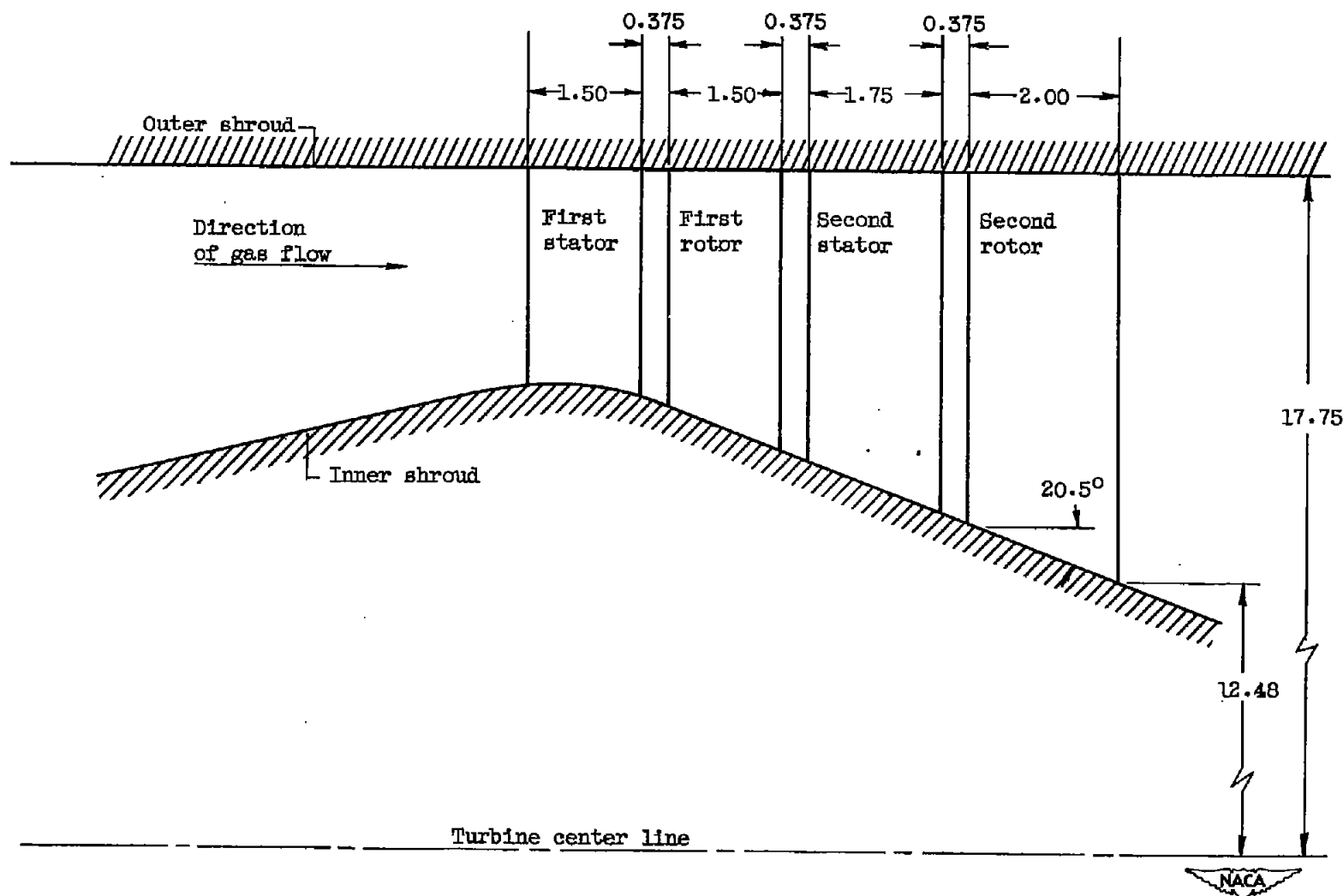


Figure 11. - Two-stage turbine configuration for increased-diameter turbine. Dimensions are in inches.